GENERALIZED DESIGN OF BAND-PASS AND OTHER FILTERS BY COMPUTER OPTIMIZATION

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Abstract

A straightforward method yields accurate equal-ripple response for band-pass, low-pass, etc., filters. The circuit can contain unequal line lengths, lumped elements, and discontinuity-effect representations. Examples include a 30-resonator band-pass design.

Exact synthesis of microwave filters is feasible when the circuit consists entirely of equal or commensurate line lengths. Filters with noncommensurate lines and lumped elements have not yielded exact, optimum synthesis, although interesting progress has been made. But most practical microwave filters do not have equal or commensurate lengths; for example, coaxial low-pass filters, waveguide band-pass filters, and comb-line filters. These have been designed by approximate methods. Results have been usually good, but errors can be quite large in wide-band band-pass filters.

In this paper, an efficient, fast numerical optimization method yielding equal-ripple response is described briefly, and then its wide applicability and power are illustrated by design examples. The range of applicability is limited only by the requirement that the filter be nondissipative, representable by an equivalent circuit, and symmetrical or antisymmetrical.

The optimization method is similar to one published by Remez[1,2] in 1934 for approximating functions by polynominals, and by Kammler in designing directional couplers.[3] Gupta[4] has recently used a similar method to design low-pass microwave filters; this is discussed later.

Figure 1a shows the desired reflection-coefficient response of an n-resonator band-pass filter. The amplitude $|\rho| = \delta$ occurs at $m = n+1$ frequencies--the band edges f_1 and f_m, and at maxima points f_2 to f_{m-1}. However, $|\rho|$ is a poor choice as a function to be made equal ripple since, for example, if $|\rho|$ is forced to equal δ at f_2 and f_3, there is no assurance that $|\rho|$ will be zero somewhere between f_2 and f_3. Complex ρ also is not suitable. The ideal function would be pure real, exhibiting alternate maxima and minima, as illustrated by ρ' in Fig. 1b. Fortunately, ρ' can be computed; let A', B', C', D' be matrix elements of the bisected half of a symmetrical, nondissipative filter. Then

$$\rho' = \frac{x}{\sqrt{1+x^2}} \; ; |\rho'| = |\rho| \tag{1}$$

where

$$x = (B'D' - A'C')/j \tag{2}$$

Because A' and D' are real and B' and C' are imaginary, x is real and both x and ρ' have the behavior of Fig. 1b. Either can be used as the response function to be made equal ripple in the optimization process.

At each frequency f_i, the error from equal-ripple response is

$$y_i = \rho_i' - (-1)^i \delta, \quad i=1 \text{ to } m \tag{3}$$

An iterative process to reduce each y_i to zero is carried out, requiring the solution of m simultaneous equations relating y_i to the corrections of the m independent parameters. After each iteration the response and errors y_i are computed again with the corrected parameters, until the errors are judged sufficiently small.

Deviations of f_2 to f_{m-1} from the true equal-ripple maxima and minima points are tested by computing ρ' at $f_i \pm \Delta f$ as well as at f_i, where Δf is chosen small compared to ripple width. Figure 2a shows f_i correctly centered at a maximum with

$$\rho'(f_i - \Delta f) = \rho'(f_i + \Delta f) < \rho'(f_i),$$

while Fig. 2b shows f_i off the maximum point. An improved value of f_i may be obtained by evaluating the coefficients of the parabola $\rho' = a + bf + cf^2$ that passes through the three points at f_i and $f_i \pm\Delta f$. At the maximum (and minimum), $d\rho'/df = 0$, and the following correction for f_i was derived.

$$(f_i)_{new} = f_i + \frac{[\rho'(f_i-\Delta f) - \rho'(f_i+\Delta f)]\Delta f}{2[\rho'(f_i-\Delta f) + \rho'(f_i+\Delta f) - 2\rho'(f_i)]} \tag{4}$$

By using Eq. 4 in the last few iterations, the program automatically finds the correct f_i, while producing the correct ripple amplitude.

Gupta's method[4], referred to earlier, differs as follows from the method of this paper. (1) His set of simultaneous equations includes equations to solve for the f_i within the pass band. Thus the general band-pass filter with specified band edges requires 2n simultaneous equations to be solved, rather than the n+1 used in this paper. (2) He treates only low-pass symmetrical filters, which have (n-1)/2 independent f_i excluding the band edge, and (n+1)/2 independent parameters, requiring n simultaneous equations compared to (n+1)/2. (3) Gupta computes his response function from the ABCD matrix of the total filter, rather than that of its bisected half.

For the iterative process to converge to a solution, the initial set of filter parameters and f_i must be reasonably good. Approximate design formulas for many microwave filter circuits have been published, and these provide adequate starting points. Also, relatively simple design approaches are available for treating new circuits. Accuracy of the initial f_i set is adequate if each f_i lies slightly within the interval between the appropriate inflection points of the correct equal-ripple response. In the case of interdigital and other filters having a symmetrical f_i distribution, the initial set of f_i values may be computed from these approximate formulas.

$$f_i = \frac{2f_o}{\pi} \tan^{-1}\left[\frac{1-u^2}{u}\right]^{1/2} \; ; i=1 \text{ to } n+1$$

where

$$u = \cos\left(\frac{\pi f_1}{2 f_o}\right) \cos\left(\frac{(i-1)\pi}{n}\right), \quad f_o = \frac{f_1 + f_{n+1}}{2} \tag{5}$$

If the response is highly unsymmetrical, as in the case of wide-band comb-line and waveguide filters, the f_i values from Eq. 5 should be adjusted judiciously or, if available, with a frequency-scale mapping function. For example, the following may be used for comb-line filters.

$$f_i = \frac{f_o}{\theta_o} \tan^{-1} \left[\frac{\tan \theta_1 + \theta_{n+1}}{2} + \left(\frac{(f_i)_{Eq. 5} - f_o}{f_o - f_1} \right) \left(\frac{\tan \theta_{n+1} - \tan \theta_1}{2} \right) \right] \tag{6}$$

As an example, consider the n=5 comb-line filter with tapped terminations in Fig. 3. The number m of critical frequencies and of independent parameters is m = n+1 = 6. The symmetrical circuit has 10 parameters: θ_o and θ_{To} (θ and θ_T at f_o), Y_1, Y_2, Y_3, Y_{12}, Y_{23}, C_1^s, C_2^s, and C_3^s. Therefore, four of these are not independent and may be preassigned--for example, let $\theta_o = 45^o$ and let

$$Y_i = (70 \text{ ohms})^{-1} - Y_{i-1, i} - Y_{i, i+1}, \quad i=1, 2, 3.$$

For n=6, one additional independent parameter is needed, which would be Y_{34}.

Figure 4 shows an interdigital filter and its equivalent circuit. Because this is an equal-line-length filter, its frequency response is symmetrical around f_o corresponding to $\theta_o = 90^o$. For n=5, there are six critical f_i, but in view of frequency symmetry, only three of these are independent. If Z_1, Z_2 and Z_3 are selected as the three independent parameters of the filter, then Z_{12} and Z_{23} may be pre-assigned any values that do not violate physical realizability. For n even, Z_{12} and n/2 stubs can be the independent parameters.

A number of comb-line designs with tapped terminations were computed by the optimization program, with initial parameters obtained from approximate formulas.[5] The final parameters are listed below for case with n=15 resonators, $w = (f_{16}-f_1)/f_o = 0.2545$, ripple VSWR = 1.10, and $\theta_o = 45^o$. Iteration was stopped when the VSWR error from exact equal ripple was within ±0.000005.

$\theta_{To} = 22.7344^o$

i	$C_{i,i+1}/\epsilon$	C_i^s(pF)
1	1.47965	6.65258
2	0.993686	5.72360
3	0.924791	5.92440
4	0.897734	5.98003
5	0.884568	5.99344
6	0.878035	5.99775
7	0.875280	5.99924
8		5.99970

The optimization program was also adapted for the interdigital band-pass filter, with Cristal's[6] approximate design formulas providing the initial parameters. An exact synthesis for this filter exists,[7] and design tables for n up to 10 are included in a report.[8] One of these designs was verified, and then additional designs outside the range of these tables were computed including an n=30 filter with 70% bandwidth. In that case the desired 1.10 VSWR ripple was achieved within ±0.000005 after only four iterations. The equivalent-circuit parameters are

I	Z(I)/R	Z(I, I+1)/R
1	1.24238	1.07326
2	1.10704	1.29738
3	0.992485	1.42889
4	0.923283	1.48244
5	0.897071	1.50883
6	0.884220	1.52357
7	0.876985	1.53252
8	0.872472	1.53833
9	0.869633	1.54223
10	0.867566	1.54491
11	0.866249	1.54680
12	0.865265	1.54811
13	0.864684	1.54897
14	0.864254	1.54945
15	0.864096	1.54962

The method's power is illustrated by design examples of 15 and 30 resonator band-pass filters. The upper limit of applicability of the Basic-language program using General Electric's Mark I time-sharing service is believed to be about 80 resonators for equal-line-length filters, and 40 resonators for the general circuit.

References

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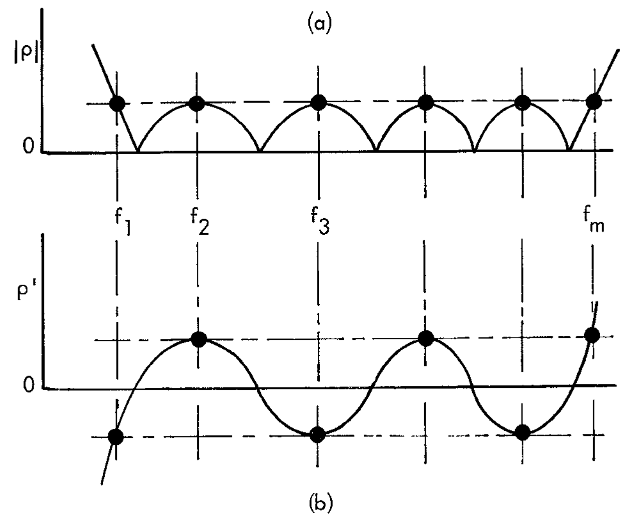

Fig. 1. Equal-ripple response functions.

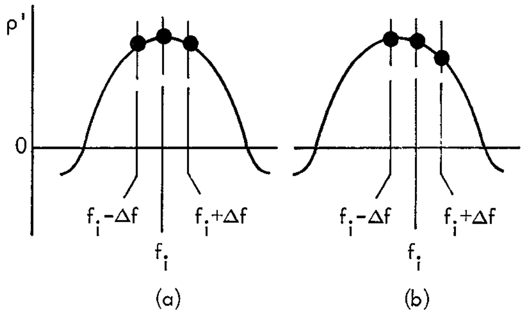

Fig. 2. f_i on maximum at (a), off maximum at (b).

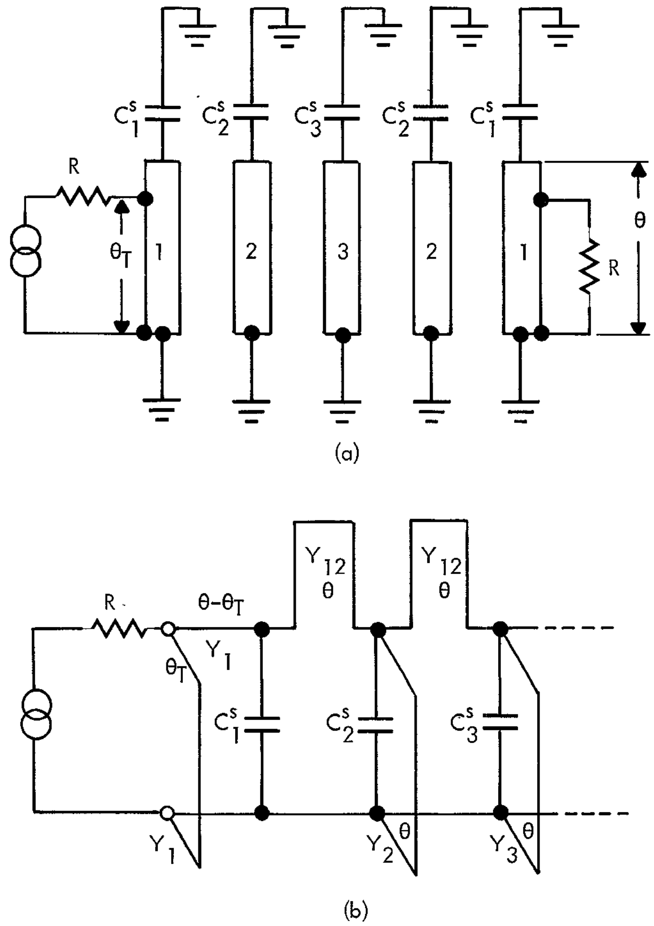

Fig. 3. Comb-line BPF with tapped terminations and its equivalent circuit.

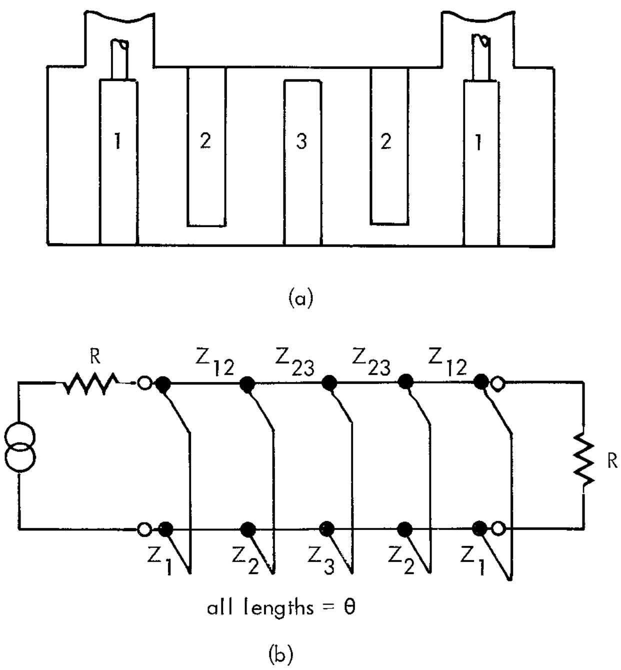

Fig. 4. n=5 interdigital BPF and its equivalent circuit.